



ANALYSIS OF STUDENTS' ERRORS IN SOLVING EQUIVALENT AND INVERSE PROPORTION PROBLEMS BASED ON THE KASTOLAN THEORY IN CLASS V STUDENTS

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Abstrak: Penelitian ini dilatarbelakangi oleh rendahnya pemahaman siswa terhadap konsep perbandingan senilai dan berbalik nilai yang menyebabkan terjadinya kesalahan dalam menyelesaikan soal matematika. Penelitian ini bertujuan untuk menganalisis jenis kesalahan siswa berdasarkan teori Kastolan. Metode yang digunakan adalah deskriptif kuantitatif. Subjek penelitian berjumlah 24 siswa kelas V-A MIS Maura El Mumtaz, dengan enam siswa dipilih secara purposive untuk dianalisis lebih mendalam. Teknik pengumpulan data meliputi tes tertulis, wawancara, dan dokumentasi. Hasil penelitian menunjukkan bahwa kesalahan konseptual merupakan kesalahan yang paling dominan dengan persentase 51,35% (kategori sedang), sedangkan kesalahan prosedural dan teknis masing-masing sebesar 24,32% (kategori rendah). Kesalahan konseptual terjadi karena siswa belum memahami dan membedakan konsep perbandingan senilai dan berbalik nilai, kesalahan prosedural disebabkan oleh ketidaktepatan langkah penyelesaian, dan kesalahan teknis terjadi karena kurangnya ketelitian serta lemahnya penguasaan operasi hitung dasar. Dengan demikian, dapat disimpulkan bahwa kesalahan siswa saling berkaitan dan berawal dari kurangnya pemahaman konsep, sehingga diperlukan pembelajaran yang lebih menekankan pada pemahaman konsep, penggunaan media yang sesuai, serta pembiasaan berpikir sistematis.

Kata kunci: Kesalahan Siswa, Perbandingan Senilai, Perbandingan Berbalik Nilai, Teori Kastolan

Abstract: This research is motivated by students' low understanding of the concept of equivalent and inverse comparisons, which causes errors in solving mathematics problems. This study aims to analyze the types of student errors based on the Kastolan theory. The method used is descriptive quantitative. The research subjects were 24 students of class V-A MIS Maura El Mumtaz, with six students selected purposively for further analysis. Data collection techniques included written tests, interviews, and documentation. The results showed that conceptual errors were the most dominant error with a percentage of 51.35% (moderate category), while procedural and technical errors each accounted for 24.32% (low category). Conceptual errors occur because students do not understand and differentiate the concepts of equivalent and inverse comparisons, procedural errors are caused by inaccurate solution steps, and technical errors occur due to a lack of accuracy and weak mastery of basic arithmetic operations. Thus, it can be concluded that student errors are interrelated and originate from a lack of conceptual understanding, so that learning is needed that emphasizes conceptual understanding, the use of appropriate media, and the habit of systematic thinking.

Keywords: student errors, direct proportion, inverse proportion, Kastolan's theory, mathematics learning

INTRODUCTION

Mathematics is a subject that plays a crucial role in developing students' logical, systematic, and critical thinking skills, starting in elementary school. According to Oktavia and Hutajulu (2022), mathematics is taught to students because: (1) it is used in all areas of life; (2) it requires appropriate mathematical skills for all fields; (3) it requires concise, robust, and clear means of communication; (4) it presents data in various models; (5) it fosters logical thinking and



thoroughness; and (6) it fosters a sense of satisfaction from solving problems. Therefore, mathematics learning is crucial for all future generations of the nation. In mathematics, students are expected not only to memorize mathematical concepts but also to understand them thoroughly. By understanding mathematical concepts, they will be able to easily apply these concepts to solve problems related to them (Rosiana, 2021).

This aligns with the Ministry of Primary and Secondary Education's (2025) mathematics learning objectives, which are to enable students to understand and apply mathematical facts, concepts, principles, operations, and relationships. However, in classroom practice, students often perceive the subject as difficult. This difficulty not only leads to students' lack of conceptual understanding but also to confusion when working on problems. When students don't fully understand what they are learning, they tend to hesitate in determining the steps to solve the problem, often resulting in errors in solving math problems.

Errors in solving math problems are common and can provide insight into students' level of understanding, as these errors not only result in inaccurate final results but also reflect obstacles in students' thinking processes. Therefore, analyzing student errors is crucial to identify the types and causes of these errors. Through error analysis, teachers can understand which parts of the material students have not yet mastered and how students process information when solving math problems (Annisa & Kartini; 2023).

One of the mathematics topics that often causes difficulty for students is comparison, particularly equivalent and inverse ratios. An equivalent ratio is a relationship between two or more quantities whereby an increase in one quantity causes the other to increase, and vice versa. Meanwhile, an inverse ratio is a relationship between two quantities whereby an increase in one quantity causes the other to decrease or become smaller (Priatna & Yuliardi et al., 2018). This material requires a strong conceptual understanding and the ability to apply it to various problem types, particularly word problems.

Consistent with this, Andrini and Sejarahi (2023) stated that students still experience difficulty understanding and solving comparison problems. To identify student errors, teachers can provide problems related to the material taught, particularly word problems. This indicates that students' difficulties lie not only in the calculation aspect, but also in conceptual understanding and the ability to translate problems into mathematical form. Furthermore, research by Selpiana and Mulbar (2025) also shows that students frequently experience misunderstandings regarding equivalent and inverse value comparisons. These errors are primarily caused by an inadequate conceptual understanding, limited mathematical reasoning skills, and low metacognitive abilities.

Ideally, students are expected to be able to distinguish between equivalent and inverse ratios, interpret word problems correctly, and apply procedures according to the concepts. However, learning that focuses too much on procedures and lacks contextualization can reinforce student misunderstandings. This suggests that mathematics instruction needs to provide space for exploration of meaning and application of concepts in various real-world situations so that student understanding does not stop at the procedural stage. Students also report experiencing difficulties



in understanding equivalent and inverse ratio word problems, including identifying known and required data, and correctly applying concepts and formulas (Selpiana & Mulbar, 2025).

Difficulties in understanding equivalent and inverse ratio material can lead to students making various types of errors in the problem-solving process. Therefore, an analytical approach is needed that can systematically identify these types of errors. One theory that can be used is the Kastolan theory. Kastolan is a theory that identifies three types of student errors: conceptual, procedural, and technical errors. Analysis of student errors will be conducted by considering indicators for each type of error experienced by students (Halawa et al., 2024). Conceptual errors arise when students cannot determine the appropriate formula to solve a problem or even fail to include a formula in the solution process. According to Rosiana (2021), students' lack of understanding of the concept of ratio causes difficulties in solving calculation problems that use the concept. Furthermore, procedural errors include inaccuracies in writing the steps to solve the problem and failure to include the final conclusion of the results obtained. Meanwhile, technical errors relate to errors in executing mathematical operations, such as multiplication, division, subtraction, and addition, as well as errors in writing variables and constants (Ndek & Suwanti, 2022). By using Kastolan's theory, student error analysis can be conducted in a more focused and systematic manner. This theory not only helps identify the types of errors but also provides insight into the sources of student errors.

Based on this description, studies examining elementary school students' errors in solving direct and inverse proportion problems using Kastolan's error analysis theory remain limited, despite the fact that these topics are conceptually challenging and frequently lead to student misconceptions. Therefore, this study aims to analyze the errors made by fifth-grade students in solving direct and inverse proportion problems based on Kastolan's theory. The study contributes to the existing literature by extending the application of Kastolan's error analysis framework to the context of elementary mathematics, a level that has received relatively little attention in previous research. Furthermore, it identifies specific patterns and sources of students' conceptual, procedural, and technical errors, providing empirical evidence that can inform the design of more targeted instructional strategies, diagnostic assessments, and remedial interventions. The findings are expected to serve as a valuable reference for teachers, curriculum developers, and researchers in improving mathematics instruction, particularly in teaching proportional reasoning at the elementary school level.

RESEARCH METHOD

The research uses a quantitative descriptive approach to describe and analyze student errors in solving comparison problems. This research was conducted at MIS Maura El Mumtaz, located in Binjai City, North Sumatra.

The subjects were all 24 students in grades V-A. The sampling technique used was purposive sampling, which involves deliberately selecting subjects based on certain criteria, specifically students who experienced errors in solving comparison problems. The sample of six students in



this study was selected for more in-depth analysis. Furthermore, the class teacher and the six students were also involved as informants in interviews to obtain more valid data.

Data collection techniques used were a written test and interviews. The written test consisted of three questions: two feasible comparison questions and one value comparison question, to measure student ability. Interviews were conducted with both teachers and students to obtain further information regarding learning difficulties and the learning process taking place in the classroom.

The data analysis technique used was descriptive analysis based on Kastolan’s theory. Kastolan’s theory classifies student errors into three types: conceptual errors, procedural errors, and technical errors. The data obtained was analyzed to identify the types and levels of student errors in solving comparison problems.

Indicators of student errors according to Kastolan's theory (Sari & Najwa, 2021 in Suharnita et al., 2024) are presented in the Table 1:

Table 1. Kastolan Error Analysis Indicators

No	Error Type	Indicator
1	Conceptual Error	Students cannot choose the correct formula or students forget the formula that must be used. The student was correct in choosing the formula but was unable to apply the formula correctly Students cannot determine the formula to answer a problem. Students use formulas, theorems, or definitions that do not comply with the prerequisite conditions for the formula to be valid.
2	Procedural Error	Students take inappropriate steps to solve problems Students cannot solve the problem down to its simplest form, so further steps need to be taken. Students are not consistent in carrying out the calculation steps. Students are unable to manipulate the steps to answer a problem.
3	Engineering Error	Students make mistakes in calculating the value of an arithmetic operation Students move constant or variable values from one step to the next.

The percentage of types of student errors is used to determine how large each type of error is. To calculate the percentage of student error types, the following formula can be used:

$$P_i = \frac{x_i}{\sum x} \times 100\%$$

Description:

- P_i = Percentage of student errors in the i-th type
- x_i = Number of errors experienced by students in the i-th type
- $\sum x$ = Total possible errors

Table 2. Criteria for Student Error Levels

No	Criteria	Error Levels
1	$0\% \leq P < 20\%$	Very Low
2	$20\% \leq P < 40\%$	Low
3	$40\% \leq P < 60\%$	Moderate
4	$60\% \leq P < 80\%$	High
5	$80\% \leq P < 100\%$	Very High

RESULT AND DISCUSSION

Based on the analysis of the instrument provided, including the material on comparing equivalent and inverse values, the following is the percentage of student errors based on the castolan stage:

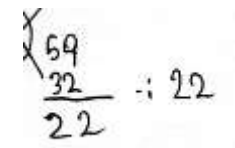
Table 3. Percentage Results by Castolan Stage

Error Type	Mistakes Made By Students			Total	Percentage	Error Rate
	On Each Question Item					
	1	2	3			
Conceptual	4	3	12	19	51,35%	Currently
Procedural	3	5	1	9	24,32%	Low
Technique	3	5	1	9	24,32%	Low
Amount	10	13	14	37	100%	

Table 3 shows that conceptual errors were the highest among students, reaching 51.35%, with a moderate error rate. This indicates that some students still made errors in formula selection or were unable to apply the formula correctly. Second place came procedural and technical errors, with these errors reaching 24.32%, with a low error rate. This means that 37.5%, or 9 out of 24 students, still made errors in solving problems and calculating the value of an arithmetic operation. Conceptual errors were most common in question number 3, with 12 students making errors, followed by question number 2 with 3 errors, and question number 1 with 4 errors. For procedural and technical errors, students made the most errors in question 2 with 5 errors, followed by question 1 with 3 errors, and question 3 with 1 error.

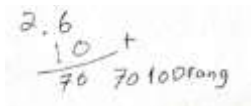
The following are the errors students make when solving comparison and inverse value problems, as analyzed based on the stages of the kastolan (reference) process:

The first type of error is conceptual errors, as seen in Figures 1, 2, and 3. Based on data analysis, conceptual errors were the most common, accounting for 51.35%. This indicates that most students still struggle to understand the basic concepts of equivalent and inverse value comparisons.



Handwritten calculation: $\frac{59}{32} = 22$

Figure 1. Conceptual error number 1



Handwritten calculation: $\frac{2.6}{10} = 70$

Figure 2. Conceptual error number 2

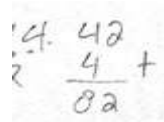


Figure 3. Conceptual error number 3

These three figures show that students made errors in selecting and using the correct concepts to solve the problems. In Figure 1, the student uses subtraction when solving the problem, instead of using the concept of equivalent comparison. In Figure 2, the student uses addition, which is inconsistent with the concept of equivalent comparison. Meanwhile, in Figure 3, the student does not understand the concept of inverse value comparison and reverts to addition. This indicates that students are unable to differentiate between types of comparisons and do not understand the relationships between quantities in the problem. The following are the results of interviews with three samples of students who made errors.

Question Number 1

P: "This is about equivalent ratios. Do you remember the concept?"

S1: "Yes, ma'am. If one increases, the other also increases."

P: "Then why did you use subtraction here?"

S1: "Because I thought it could be subtracted, ma'am."

Question Number 2

P: "Do you know how to do equivalent ratios?"

S2: "I still don't understand, ma'am."

P: "Here you're using addition, why?"

S2: "I see the numbers are increasing, so I thought they were added."

P: "So you're just looking at the change in numbers, huh?"

S2: "Yes, ma'am."

Question Number 3

P: "What made you use addition in question 3?"

S3: "It's a story, so I don't understand what it means."

P: "So you didn't understand the problem, so you just added it?"

S3: "Yes, ma'am."

Based on the interview results, conceptual errors occurred because students did not fully understand the meaning of comparison, particularly in distinguishing between equivalent and inverse ratios. Students tended to use basic arithmetic operations without considering the relationships between quantities in the problem. This indicates that students were unable to construct mathematical models from word problems, resulting in strategies that did not align with the concepts.

Furthermore, these errors indicate that students' understanding was still shallow, procedural, rather than conceptual. Students did not understand the "why" of a method, but simply tried to use methods they considered easier. These results align with research by Lawuna (2022), which showed that errors in comparison material were dominated by conceptual errors because students were unable to interpret the relationships between variables in the problem.

Therefore, conceptual understanding is a key factor in determining student success in solving problems.

Data analysis revealed that students made relatively low procedural errors, with a percentage of 24.32%.

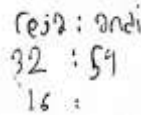


Figure 4. Procedural Error Number 1

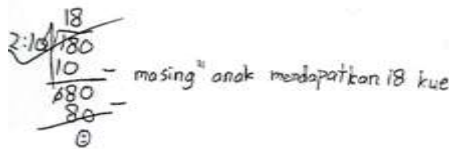


Figure 5. Procedural Error Number 2

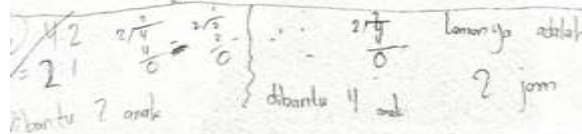


Figure 6. Procedural Error Number 3

Based on these three images, it appears that students made errors in the problem-solving steps. In Figure 4, the student failed to complete the calculation to its simplest form, leaving the solution incomplete. In Figure 5, the student performed an incorrect step, directly dividing 180 by 10. The correct solution would have been to first multiply 6 children by 30 cookies to find the total number of cookies before dividing the total number of cookies among the 10 children. Meanwhile, in Figure 6, the student performed a ratio of 4:2, which should have been $\frac{2}{4} \times 2$, which would have resulted in a ratio of 2 hours for the additional 4 children.

This indicates that students do not understand the correct procedures for solving equivalent ratios and inverse ratios. The following are the results of interviews with three students who made procedural errors.

Question Number 1

P: "Please explain your steps in solving this problem."

S1: "I'll just calculate it, ma'am, using the numbers I have."

P: "Did you determine first whether this is an equivalent ratio?"

S1: "No, ma'am. I'll just calculate it."

P: "If it's an equivalent ratio, what are the steps?"

S1: "I don't remember, ma'am."

P: "Why don't you continue to the simplest form?"

S1: "I think that's enough."

Question Number 2

P: "In this problem, why did you divide right away?"

S3: "I thought you just divided, ma'am."

P: "Did you determine the relationship between the values first?"

S3: "No, ma'am."

P: "If it's an equivalent ratio, what should you do first?"

S3: "You should compare first, ma'am... but I forgot how."

P: "So you're not sure about the steps?"

S3: "Yes, ma'am, I'm still confused."

Question Number 3

P: "What kind of comparison do you think this question is?"

S5: "I don't know, ma'am."

P: "It's actually an inverse value. Do you know what that means?"

S5: "I don't understand, ma'am."

P: "If it's an inverse value, are the steps the same as equivalent?"

S5: "It seems different, but I don't know the difference."

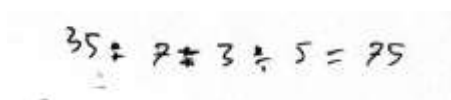
P: "Is that why your steps are out of order?"

S5: "Yes, ma'am, I'm not sure where to start."

Based on interview results, procedural errors occurred because students did not understand the systematic steps for solving comparison problems. Students did not begin by determining the type of comparison, resulting in undirected steps. Furthermore, students were not accustomed to arranging steps sequentially, from determining the relationship between quantities to completing the calculation.

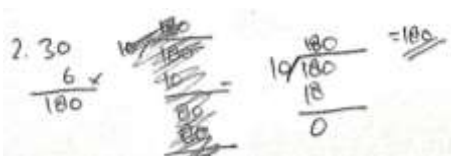
These errors indicate that students were unable to accurately connect concepts and procedures. Although some students had a preliminary understanding, they still struggled to apply it to the solution steps. This aligns with research by Inggriani et al. (2024), which found that students experienced errors at the transformation and process stages due to their inability to systematically organize the solution steps. Students did not understand concepts and procedures, resulting in inaccurate and inconsistent steps. Therefore, students' procedural understanding still needs to be strengthened through more structured learning.

Based on the analysis, the percentage of technical errors made by students was also relatively low, with a percentage of 24.32%.



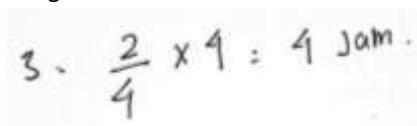
A handwritten mathematical expression showing a sequence of divisions: $35 \div 7 \div 3 \div 5 = 75$. The expression is written in a single line with no intermediate results or grouping.

Figure 7. Technical Error Number 1



Three handwritten calculations are shown. The first is $2. 30 \div 6 = 180$. The second is $180 \div 10 = 18$. The third is $18 \div 0 = 180$. The calculations are written in a single line with no intermediate results or grouping.

Figure 8. Technical Error Number 2



A handwritten mathematical expression showing a multiplication: $3. \frac{2}{4} \times 4 = 4 \text{ jam.}$ The expression is written in a single line with no intermediate results or grouping.

Figure 9. Technical Error Number 3



Based on these three images, it appears that students made errors in arithmetic operations. In Figure 7, the student made an error in division, either calculating or simplifying the division to its simplest form. In Figure 8, the student made an error in building division. When using building division, the student should first multiply 10 by 1. Then, if the result is 80, the student should multiply 10 by 8 to get 80. The final answer is that each student will receive 18 cookies. In Figure 9, the student made an error in multiplication and division, where the student incorrectly performed both operations. The student received a score of 4 hours. The student should have received a score of 2 hours if the multiplication and division operations were correct. The following are the results of interviews with three students who made technical errors.

Question Number 1

P: "You already know this is an equivalent ratio, but why is the result wrong?"

S2: "I miscalculated, ma'am."

P: "Where?"

S2: "It's correct, I wasn't careful enough."

P: "Did you double-check the result?"

S2: "No, ma'am."

P: "But if you recalculated it, it would be different, right?"

S2: "Yes, ma'am."

Question Number 2

P: "Please explain how you calculated it."

S4: "I immediately divided it using the stacking method, ma'am."

P: "Are you sure the division steps were correct?"

S4: "I'm sure, ma'am."

P: "But why is the result still wrong? Do you usually double-check?"

S4: "I still don't know multiplication and I wasn't careful enough, ma'am."

Question Number 6

P: "In this problem, did you already know it was an inverse?"

S6: "Not really, ma'am."

P: "Why did the result take 4 hours?"

S6: "I made a mistake with multiplication and division."

P: "Are you good at division and multiplication yet?"

S6: "Not very good at it, ma'am, that's why it took me 4 hours to get the results."

Based on the interview findings, the technical errors made by students were primarily caused by a lack of accuracy in performing arithmetic operations and insufficient mastery of basic mathematical skills, particularly multiplication and division. Although several students demonstrated an understanding of the underlying concepts of ratio, they frequently made computational mistakes during the calculation process, resulting in incorrect final answers. In addition, many students tended not to recheck their solutions before submitting them, allowing avoidable calculation errors to remain undetected. These findings indicate that procedural fluency and computational accuracy remain essential components in solving proportional reasoning



problems. This result is consistent with the findings of Syam (2025), who reported that technical errors commonly arise from students' limited mastery of basic arithmetic operations and their lack of carefulness during the calculation process. Similar findings have also been reported by Wijaya et al. (2020), who emphasized that weaknesses in numerical computation often hinder students' success in solving mathematical word problems, even when conceptual understanding has begun to develop.

The analysis of students' responses across the three test items revealed varying levels of error. Ten students (27.02%) made errors on Question 1 and thirteen students (35.13%) on Question 2, both of which fall into the low error category. However, the highest error rate occurred on Question 3, with 37.83% of students answering incorrectly. This finding suggests that inverse proportion is substantially more difficult for elementary students than equivalent (direct) proportion. Unlike direct proportion, inverse proportion requires students to understand the reciprocal relationship between two variables, a concept that is often abstract and cognitively demanding. Consequently, many students incorrectly applied the procedures used for direct proportion when solving inverse proportion problems. These findings support previous studies indicating that inverse proportion represents one of the most challenging proportional reasoning concepts for elementary school students because it requires flexible reasoning rather than reliance on memorized algorithms (Lamon, 2020; OECD, 2023).

Furthermore, the analysis demonstrates that students' errors were not independent but rather interconnected. Conceptual errors frequently led to procedural errors, which subsequently resulted in technical or computational mistakes. Students who misunderstood the relationship between quantities were more likely to select inappropriate solution strategies and perform incorrect calculations. This hierarchical pattern of errors is consistent with Kastolan's error theory, which classifies mathematical errors into conceptual, procedural, and technical categories that influence one another during the problem-solving process. Similar conclusions have been reported by Nurjanah et al. (2022), who found that students with weak conceptual understanding are significantly more likely to commit procedural and computational errors when solving ratio and proportion problems.

The interview with the classroom teacher further reinforced these findings. According to the teacher, students exhibited varying levels of mathematical ability. While some students quickly understood the concept of ratio, others still struggled with distinguishing between equivalent and inverse proportions and continued to experience difficulties with multiplication and division. The teacher also explained that students often encountered problems when solving word problems because they were not accustomed to relating mathematical concepts to real-life situations. Moreover, classroom instruction was still dominated by teacher explanations followed by routine exercises, providing limited opportunities for students to construct conceptual understanding through meaningful learning experiences. This finding is consistent with research highlighting that contextual and student-centered learning environments significantly improve proportional reasoning and mathematical problem-solving abilities by encouraging students to connect abstract mathematical ideas with authentic situations (Van Dooren et al., 2021; OECD, 2023).



Overall, the findings provide a comprehensive picture of the types and causes of students' errors in solving equivalent and inverse proportion problems. The results have important pedagogical implications, suggesting that mathematics instruction should place greater emphasis on strengthening conceptual understanding before introducing procedural algorithms. Teachers should also provide contextual learning activities, encourage students to explain their reasoning, and cultivate the habit of reviewing completed solutions to minimize technical errors. Although this study is limited by its relatively small sample size and its focus on a single mathematical topic, its major strength lies in the integration of written test analysis and interview data. This triangulation approach enables a deeper understanding of not only the errors students made but also the underlying cognitive factors contributing to those errors, thereby providing valuable information for improving mathematics instruction at the elementary school level.

CONCLUSIONS AND RECOMMENDATIONS

Based on the research results, it can be concluded that students still make errors in solving equivalent and inverse ratio problems, including conceptual, procedural, and technical errors. Conceptual errors were the most prevalent, accounting for 51.35%, categorized as moderate, while procedural and technical errors each accounted for 24.32%, categorized as low. The predominance of conceptual errors indicates that the primary problem lies not only in calculation ability, but also in students' basic understanding of the concept of ratio itself. Students' inability to distinguish between equivalent and inverse ratios and to understand the relationships between quantities leads to errors continuing into the procedural and calculation stages, indicating a strong correlation between these types of errors.

These results indicate that the current learning process has not fully helped students grasp the concept. Therefore, teachers need to develop learning that focuses not only on lectures and practice problems, but also on using more varied learning media, such as visual aids or concrete media relevant to students' lives, to facilitate comprehension of the concept of ratio. Furthermore, students need to be accustomed to thinking more systematically and carefully when solving problems, so that they not only understand the concept but also apply it correctly.

REFERENCES

- Andarini, H., & Riwayati, S. (2023). Analisis kesalahan siswa dalam menyelesaikan soal cerita perbandingan berdasarkan prosedur Newman. *Lemma: Letters of Mathematics Education*, 9(2), 82–94. <https://doi.org/10.22202/jl.2022.v9i2.5404>.
- Anisa, U. I., & Kartini, K. (2023). Analisis Kesalahan Siswa Berdasarkan Teori Kesalahan Kastolan dalam Menyelesaikan Soal Relasi dan Fungsi Kelas VIII SMP IT Bangkinang. *Edumatica: Jurnal Pendidikan Matematika*, 13(2), 172-180. <https://doi.org/10.22437/edumatica.v13i02.19473>.
- Halawa, S., Mailani, E., Lubis, W., Simanjuntak, S., & Prawijaya, S. (2024). Analisis kesalahan siswa dalam menyelesaikan soal cerita menurut teori Kastolan pada materi pecahan di SD kelas V T.A 2023/2024. *Jurnal Guru Kita PGSD*, 8(3), 559–566. <https://doi.org/10.24114/jgk.v8i3.58475>



- Inggriyani, P., & El Hakim, L. (2024). Analisis Kesalahan Siswa dalam Menyelesaikan Soal Cerita Materi Perbandingan dengan Prosedur Newman di SMPN 236 Jakarta. *Jurnal Riset Pendidikan Matematika Jakarta*, 6(2), 43-56. <https://doi.org/10.21009/jrpmj.v6i2.47931>.
- Kemendikdasmen. (2025). *Mata Pelajaran Matematika Fase A-F dan Fase F Tingkat Lanjut*
- Lamon, S. J. (2020). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers* (4th ed.). Routledge. <https://doi.org/10.4324/9780429264145>
- Lawuna, B. (2022). Analisis Kemampuan Siswa Dalam Menyelesaikan Soal Cerita Pada Materi Perbandingan Senilai Dan Berbalik Nilai Di Kelas Viii Smp Swasta Kristen Bnkp Mazino Tahun Pembelajaran 2021/2022. *FAGURU: Jurnal Ilmiah Mahasiswa Keguruan*, 1(1), 18-27. <https://doi.org/10.57094/faguru.v1i1.498>.
- Ndek, K. Y., & Suwanti, V. (2022). Analisis Kesalahan Siswa dalam Menyelesaikan Soal Cerita Persamaan Linear Satu Variabel Berdasarkan TeoriKastolan. *JRPM (Jurnal Review Pembelajaran Matematika)*, 7(1), 89-101. <https://doi.org/10.15642/jrpm.2022.7.1.89-101>
- Nurjanah, S., Mulyono, & Rochmad. (2022). Analysis of students' errors in solving ratio and proportion problems based on Kastolan's theory. *Journal of Mathematics Education Research*, 11(2), 145–156.
- OECD. (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*. OECD Publishing. <https://doi.org/10.1787/53f23881-en>
- Oktavia, R., & Hutajulu, M. (2022). Analisis kesalahan siswa dalam menyelesaikan soal cerita perbandingan. *JPMI (Jurnal Pembelajaran Matematika Inovatif)*, 5(1), 105-112.
- Priatna, N., & Yuliardi, R. (2018). *Pembelajaran Matematika. Bandung: PT Remaja*
- Rosiana, S. (2021). Hubungan Pemahaman Konsep Perbandingan Senilai Dan Berbalik Nilai Terhadap Perhitungan Suhu Pada Fisika Kelas Vii Smp. *Teaching: Jurnal Inovasi Keguruan Dan Ilmu Pendidikan*, 1 (4). 350-356.
- Selpiana, S., & Mulbar, U. (2025). Literature Review: Misconceptions of 7th Grade Students on the Concepts of Direct and Inverse Proportion. *OMEGA: Jurnal Keilmuan Pendidikan Matematika*, 4(3), 208-217. <https://doi.org/10.47662/jkpm.v4i3.1084>.
- Suharnita, E., Kartini, K., & Saragih, S. (2024). Analisis kesalahan peserta didik pada materi luas permukaan bangun ruang sisi datar berdasarkan teori Kastolan. *JIPIS (Jurnal Ilmu Pendidikan dan Ilmu Sosial)*, 33(1), 1–12. <https://doi.org/10.33592/jipis.v33i1.4581>.
- Syam, M. A., Istifani, N., Asriani, A., Syariah, S., & Fajriah, N. A. (2025). Analisis kesalahan siswa dalam menyelesaikan soal operasi bilangan. *PENDAS: Jurnal Ilmiah Pendidikan Dasar*, 10(3), 862-872. <https://doi.org/10.23969/jp.v10i3.29914>
- Syam, A. (2025). Analysis of students' mathematical errors based on Kastolan's theory in solving arithmetic problems. *Journal of Mathematics Learning and Education*, 10(1), 34–47.
- Van Dooren, W., De Bock, D., & Verschaffel, L. (2021). From proportional reasoning to mathematical modeling: Challenges and instructional implications. *ZDM–Mathematics Education*, 53(1), 123–136. <https://doi.org/10.1007/s11858-020-01188-3>



Wijaya, A., Van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2020). Difficulties in solving contextual mathematics problems: A cross-national analysis. *The Mathematics Enthusiast*, 17(1–3), 255–276.